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MEXT

Undergraduate Natural Sciences A

Mathematics B

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Embassy recommendation 2016–2019

12 major problems / 47 questions Mathematics volume

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Official public examination papers, 2016–2019

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2017 Mathematics B / 3

Official question PDF: page 3. Read alongside the original paper.

For the intersection of two perpendicular cylinders, a horizontal slice is a square. Derive that square from both cylinder inequalities before integrating; two circular-looking constraints do not make a circular slice.

3(1) A square horizontal section

Solution

Fix $z = t$. The two cylinder inequalities become

$$x^2 \le r^2 - t^2, \qquad y^2 \le r^2 - t^2.$$

For $-r \le t \le r$, let $s = \sqrt{r^2 - t^2}$. The permitted points form the square

$$-s \le x \le s, \qquad -s \le y \le s.$$

Its side length is $2s$, so

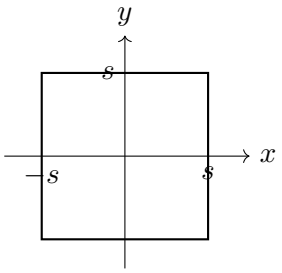
$$A(t) = (2s)^2 = 4(r^2 - t^2).$$

Answer $A(t) = 4(r^2 - t^2)$.

Explanation

Intersection, not union Each point must satisfy both bounds. One cylinder restricts x and the other restricts y, so the common section is their Cartesian-product square.

A normalized section The sketch uses $s = 1$. Its four corners all satisfy both cylinder bounds.



A diagnostic corner The point (s, s) would fail the disk inequality $x^2 + y^2 \le s^2$, but it satisfies the actual pair of separate bounds. Replacing them by a disk is therefore not an innocent simplification; it removes valid points.

3(2) Volume of the common solid

Solution

Integrate the square area over its full vertical range:

$$V = \int_{-r}^{r} 4(r^2 - t^2)\,dt.$$

By symmetry and a polynomial antiderivative,

$$V = 8\int_{0}^{r} (r^2 - t^2)\,dt = 8\left[r^2 t - \frac{t^3}{3}\right]_0^r = 8\left(r^3 - \frac{r^3}{3}\right) = \frac{16}{3}r^3.$$

Answer $V = 16r^3/3$.

Explanation

A geometric upper bound The intersection lies inside a cube of side $2r$, whose volume is $8r^3$. The answer is two-thirds of that cube volume, consistent with the sections shrinking to zero at the top and bottom.

Track the symmetry factor The section formula already contains 4 from the square's side length. Restricting the integral to $0 \le t \le r$ adds a separate factor 2. Confusing those two factors commonly halves the volume.

What the question tests The integral is elementary once the right section is identified. A three-dimensional solid with curved boundaries does not automatically require cylindrical coordinates or a difficult multiple integral. Here the common z^2 in both inequalities suggests horizontal slices immediately.

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Undergraduate Natural Sciences A

Physics

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20 sections / 92 questions Physics volume

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2019 Physics / 5

Official question PDF: page 12–13. Read alongside the original paper.

The reflected rays differ both in propagation phase and in reflection phase. Compare them at a common wavefront: using the entire internal round-trip length without subtracting the phase already accumulated by the other ray gives the wrong oblique-incidence condition.

5(1) Wavelength inside the film

Solution

A stationary interface preserves the light frequency f. The speed in the film is c/n, while the wavelength in air is $\lambda = c/f$ under the stated air-index approximation. Therefore

$$\lambda_{\text{film}} = \frac{c/n}{f} = \frac{\lambda}{n}.$$

Answer (b), $\lambda_{\text{film}} = \lambda/n$.

Explanation

Frequency continuity The transmitted oscillation is driven at the same temporal frequency as the incident light. The wavelength changes because propagation speed changes.

Higher index means shorter wavelength For $n > 1$, the speed and wavelength in the film are smaller than in air. Multiplication by n would give the opposite trend.

Use the correct wavelength in a phase calculation A geometric path difference inside the film must be compared with λ/n. Alternatively, multiply that geometric distance by n to obtain an optical path difference and compare with the air wavelength λ.

5(2) Relation between incident and refracted angles

Solution

Both angles are measured from the surface normal. Snell's law with air index 1 and film index n gives

$$1 \cdot \sin\theta = n \sin\phi.$$

Answer (d), $\sin\theta = n\sin\phi$.

Explanation

Normal-based angles The normal is perpendicular to the film surface. Measuring from the surface instead would require complementary angles and would turn sines into cosines.

Bending direction For $n > 1$, $\sin\phi = \sin\theta/n$ and hence $\phi < \theta$ for ordinary incidence angles. This matches bending toward the normal.

Same rule at the second surface At the film-to-air exit, parallel surfaces restore the original exterior propagation angle for a transmitted ray. The internal angle remains the one used in the film's propagation-phase calculation.

5(3) Constructive interference of the reflected rays

Solution

The ray reflected at the air-to-film boundary acquires a phase reversal of π. Reflection from film to air has no such reversal. Thus the two reflected rays have one relative reflection-phase reversal. Using the common-wavefront construction denoted by Q in the original paper, the extra geometric propagation length inside the film is $QB + BC$. Constructive interference requires an odd half-wavelength propagation difference to compensate for the reflection reversal:

$$QB + BC = \left(m + \frac{1}{2}\right)\lambda_{\text{film}} = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}, \qquad m = 0, 1, 2, \ldots.$$

Answer (a), $QB + BC = (m + \frac{1}{2})\lambda/n$.

Explanation

One phase reversal changes the condition With no relative reflection reversal, an integer wavelength difference would be constructive. The extra π here changes the required propagation difference to a half-integer number of wavelengths.

Why not simply AB plus BC The directly reflected ray also accumulates phase before comparison. Q accounts for this shared incident-wavefront phase, so only the remaining internal path difference belongs in the interference condition.

Geometric versus optical path Multiplying by n gives $n(QB + BC) = (m + \frac{1}{2})\lambda$. Both forms express the same phase difference, provided the path and wavelength conventions are kept consistent.

5(4) Film thickness for constructive reflection

Solution

The two internal legs each have length $d/\cos\phi$. The horizontal separation of their surface endpoints is $2d\tan\phi$. Subtracting the common-wavefront projection gives the effective internal path difference

$$\begin{aligned} QB + BC &= \frac{2d}{\cos\phi} - (2d\tan\phi)\sin\phi \\ &= \frac{2d(1-\sin^2\phi)}{\cos\phi} = 2d\cos\phi. \end{aligned}$$

Insert this into the preceding condition:

$$2d\cos\phi = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}.$$

Hence

$$d = \frac{(2m+1)\lambda}{4n\cos\phi}.$$

Answer (b), $d = (2m+1)\lambda/(4n\cos\phi)$.

Explanation

Projection changes reciprocal cosine to cosine The actual internal round trip is $2d/\cos\phi$, but it is not the phase-comparison path difference. Subtracting the reference-ray projection is essential.

Normal-incidence check At $\phi = 0$ the smallest positive thickness is $\lambda/(4n)$, the familiar quarter-wave condition for a film with one relative reflection reversal.

Track the same order For fixed m, λ and n, the required thickness increases as $\cos\phi$ decreases. The nonnegative integer labels successive constructive-reflection thicknesses; it does not alter the single reflection-phase reversal.

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Undergraduate Natural Sciences A

Chemistry

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Embassy recommendation 2016–2019

28 major problems / 156 answer fields Chemistry volume

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2019 Chemistry / VI

Official question PDF: page 6. Read alongside the original paper.

Acid–base extraction changes the charge of a compound and hence the layer in which it is preferentially retained. Keep the extracted ion distinct from the neutral compound recovered after the second reagent is added.

VI(1) Recovering aniline from the acidic layer

Solution

Aniline accepts a proton from hydrochloric acid:

$$C_6H_5NH_2 + H^+ \longrightarrow C_6H_5NH_3{}^+.$$

The resulting anilinium ion is retained in the aqueous layer. Adding sodium hydroxide removes that proton:

$$C_6H_5NH_3{}^+ + OH^- \longrightarrow C_6H_5NH_2 + H_2O.$$

The regenerated neutral aniline can then be extracted into ether.

Answer (3), aniline.

Explanation

Track two chemical forms The first aqueous layer contains the protonated form; the final ether layer contains the free amine. Reporting anilinium chloride as the recovered organic compound would stop the sequence one step too early.

Why hydrochloric acid selects the amine The nitrogen lone pair can accept a proton. Under this extraction scheme the other listed compounds do not become comparable water-soluble cations, so the amine is separated from them.

Extraction is an equilibrium separation Neutral and ionic forms have different preferences for the two solvents. The usual exam model treats this redistribution as an effective separation; it is not a claim that a single extraction gives literally zero solute in the other layer.

VI(2) Bicarbonate separates the carboxylic acid

Solution

Benzoic acid reacts with bicarbonate to form benzoate, carbon dioxide and water:

$$C_6H_5COOH + HCO_3{}^- \longrightarrow C_6H_5COO^- + CO_2 + H_2O.$$

Benzoate enters the aqueous phase. Acidifying this separated phase reverses its ionisation:

$$C_6H_5COO^- + H^+ \longrightarrow C_6H_5COOH.$$

The neutral benzoic acid is then recovered in ether.

Answer (1), benzoic acid.

Explanation

Bicarbonate is deliberately the weaker base It deprotonates the carboxylic acid effectively under these conditions, but does not extract phenol in the same way. Replacing it with a strong base at this stage would remove both acidic compounds and lose the intended separation.

Gas formation helps drive the reaction The initial proton transfer gives carbonic acid, which yields carbon dioxide and water. The written net equation makes both the acid–base change and the gas evolution visible.

Do not confuse salt and free acid The water-soluble intermediate is a benzoate salt. The answer requested after acidification and ether extraction is the parent acid, with $-\mathrm{COOH}$, not $-\mathrm{COO}-$.

VI(3) Strong base extracts phenol

Solution

After the carboxylic acid has been removed, sodium hydroxide converts phenol into phenoxide:

$$\mathrm{C_6H_5OH + OH^- \longrightarrow C_6H_5O^- + H_2O.}$$

This ion is transferred to the aqueous phase. Subsequent acidification regenerates neutral phenol:

$$\mathrm{C_6H_5O^- + H^+ \longrightarrow C_6H_5OH.}$$

The final ether extract therefore contains phenol.

Answer (2), phenol.

Explanation

The sequence distinguishes two kinds of acid Phenol remains after bicarbonate treatment but reacts with the stronger base. These observations are compatible: a compound need not react to the same extent with every base in order to be acidic.

Charge supplies the useful solubility change The neutral aromatic compound favours the organic solvent more strongly than its ionic salt does. Merely saying that phenol is soluble in one solvent misses the reason the reagent changes its layer.

The isolated aqueous layer must be treated Acid is added to the phenoxide-containing layer after it has been separated. Acidifying the original mixture without separation would regenerate the compounds together and would not accomplish the stated purification.

VI(4) The compound left in ether

Solution

Aniline has been removed by acid, benzoic acid by bicarbonate and phenol by hydroxide. Nitrobenzene remains neutral in this sequence and is retained in the final ether layer.

Answer (4), nitrobenzene.

Explanation

A nitrogen atom does not always imply an amine The nitrogen in a nitro group has a different bonding arrangement from that in $-NH_2$. Nitrobenzene therefore does not follow the protonation-and-extraction behaviour of aniline.

Elimination gives a complete material map The extracted fractions correspond in order to aniline, benzoic acid and phenol. Accounting for these three leaves exactly the fourth listed compound; none should appear as the main recovered compound in two different fractions.

The conclusion is specific to these reagents Remaining unchanged in an acid–base extraction does not mean nitrobenzene is chemically inert. Its nitro group can undergo a redox transformation, as the final part of this problem uses.

VI(5) Comparing the acidic compounds

Solution

Benzoic acid is the strongest acid among the four compounds. Its conjugate base is the benzoate ion:

$$C_6H_5COOH \rightleftharpoons H^+ + C_6H_5COO^-.$$

The negative charge of the carboxylate group is shared between two oxygen atoms through equivalent resonance contributors. This stabilisation makes proton loss more favourable than in phenol.

Answer (1), benzoic acid.

Explanation

Compare conjugate bases Both benzoate and phenoxide are resonance-stabilised, so saying only that one has resonance is insufficient. In a carboxylate the principal equivalent contributors keep the negative charge on oxygen atoms; the phenoxide comparison involves non-equivalent contributors extending into the ring.

The extraction results are an independent clue Bicarbonate removes benzoic acid but not phenol, whereas hydroxide removes phenol. That order supplies experimental evidence within the problem for the difference in acid strength.

Do not confuse acid strength with concentration The question compares the tendency of the molecular species to donate a proton under comparable conditions. The mass used or the volume of solvent is not an intrinsic measure of acid strength.

VI(6) Reduction changes nitrobenzene into aniline

Solution

Reduction converts an aromatic nitro group into an amino group without removing the benzene ring. The redox change can be represented by the half-equation

$$\mathrm{C_6H_5NO_2 + 6\,H^+ + 6\,e^- \longrightarrow C_6H_5NH_2 + 2\,H_2O.}$$

Thus X is nitrobenzene and Y is aniline.

Answer X: (4); Y: (3).

Explanation

Check atoms and charge Both sides contain six carbons and one nitrogen. The two oxygen atoms appear in two water molecules; the six protons and six electrons balance both hydrogen and charge.

Reduction is not the extraction step Tin provides a route for changing the functional group. Acid–base extraction only interconverts protonated and unprotonated forms; it does not turn a nitro group into an amino group.

The isolated form depends on work-up In an acidic reduction mixture the amine may be present as anilinium. The compound identification is aniline after neutralisation, consistent with the free compound named in the options.