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MEXT

College of Technology (KOSEN)

Mathematics Worked solutions

Embassy recommendation 2016–2019

12 major problems / 73 questions

Question text is not included. Use with the official papers.
Official public examination papers, 2016–2019

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Part 9　2018 / 3 / A logarithmic curve and its tangent region

The maximum is worth six marks, and the tangent and area are worth seven each. The main geometric issue is to identify the closed region: the curve meets the axis at $x = 1$, whereas the tangent passes through the origin.

3(1) Determine the global maximum

Solution

Differentiating $f(x) = \ln x/x$ on $x > 0$ gives

$$f'(x) = \frac{1 - \ln x}{x^2}.$$

The denominator is positive. The numerator is positive for $0 < x < e$, zero at e, and negative for $x > e$. Therefore f increases up to e and decreases after it, giving a global maximum

$$M = f(e) = \frac{1}{e}.$$

Ans. $M = 1/e$, attained at $x = e$.

Explanation

A stationary point needs a sign test　Solving $f'(x) = 0$ finds a candidate. The change from positive to negative derivative proves that it is a maximum over the whole positive domain.

Endpoint behavior　As $x \to 0^+$, the function tends to $-\infty$; as $x \to +\infty$, it tends to zero from above. These limits agree with the rise-and-fall picture.

Location versus value　The question asks for the maximum value, so e alone is not the answer. It is the input at which the value $1/e$ occurs.

3(2) A tangent through a point outside the curve

Solution

At a contact point $a > 0$, the tangent is

$$y = f'(a)(x - a) + f(a).$$

Its intercept is

$$f(a) - af'(a) = \frac{\ln a}{a} - \frac{1 - \ln a}{a} = \frac{2\ln a - 1}{a}.$$

For the line to pass through the origin, this intercept must vanish. Thus $\ln a = 1/2$, so $a = \sqrt{e}$, and

$$f'(a) = \frac{1 - 1/2}{e} = \frac{1}{2e}.$$

Ans. $y = \dfrac{x}{2e}$, with contact point $\left(\sqrt{e}, \dfrac{1}{2\sqrt{e}}\right)$.

Explanation

Why the origin is not a contact point The function is defined only for $x > 0$. The origin lies on the tangent, not on the curve, so it cannot be substituted as the point of tangency.

An equivalent condition The slope from the origin to the contact point is $f(a)/a$. Tangency requires $f'(a) = f(a)/a$, which gives the same equation $1 - \ln a = \ln a$.

Uniqueness The logarithm is strictly increasing on positive numbers, so $\ln a = 1/2$ determines exactly one positive contact point.

Check both contact conditions At $x = \sqrt{e}$, the line and curve both equal $1/(2\sqrt{e})$; their derivatives both equal $1/(2e)$. Equal heights alone would not establish tangency.

3(3) Split the bounded region at the axis crossing

Solution

The curve meets the x-axis where $\ln x = 0$, namely at $x = 1$. The bounded region runs along the axis from zero to one, then along the curve to the contact point $a = \sqrt{e}$, and back along the tangent.

On $1 \le x \le \sqrt{e}$, the tangent lies above the curve. To verify this, let

$$h(x) = x^2 - 2e \ln x, \qquad h'(x) = \frac{2(x^2 - e)}{x}.$$

The function h decreases to $h(\sqrt{e}) = 0$, so $h(x) \ge 0$ on that interval; this is equivalent to $x/(2e) \ge \ln x/x$. The area is therefore

$$S = \int_0^{\sqrt{e}} \frac{x}{2e}\, dx - \int_1^{\sqrt{e}} \frac{\ln x}{x}\, dx.$$

The first integral is $e/(4e) = 1/4$. For the second,

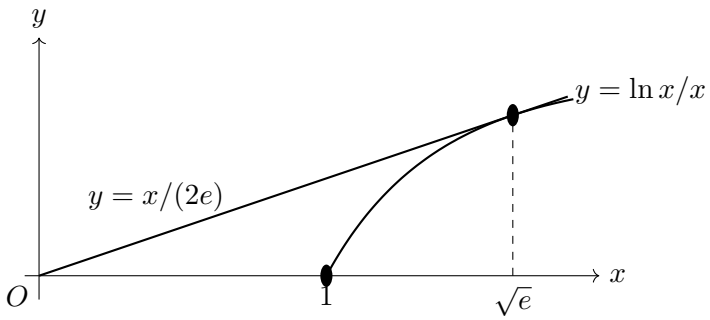
$$\left[\frac{(\ln x)^2}{2}\right]_1^{\sqrt{e}} = \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}.$$

Hence $S = 1/4 - 1/8 = 1/8$.

Ans. $S = 1/8$.

Explanation

A picture of the integration bounds The axis contributes only between zero and one; the curve then forms the lower boundary up to the contact point. The tangent is the upper boundary.



Why not integrate the curve from zero The curve is below the axis for $0 < x < 1$, and is not a boundary of the specified closed region there. Extending its area integral to zero would describe a different and divergent region.

Geometric check The triangle under the tangent has base $\sqrt{e}$ and height $1/(2\sqrt{e})$, so its area is $1/4$. The desired area must lie between zero and $1/4$; the calculated $1/8$ does.

What must accompany an area formula Identify the intersections, determine which boundary is above, and then choose the bounds. The tangent touches the curve without crossing it, so taking an absolute value after an arbitrary integral is not a substitute for this geometric check.

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Part 20 2018 / 6 / Magnetic field strength is not flux density

Each part is worth five marks. The requested unit is $\mathrm{A\,m^{-1}}$, so calculate magnetic field strength H. Magnetic flux density B is a different quantity, measured in tesla. Use the supplied $\pi \simeq 3.14$ and $\sqrt{3} \simeq 1.73$.

6(1) Add the wire fields as vectors

Solution

Each long straight wire gives magnitude

$$H_0 = \frac{I}{2\pi r} = \frac{4.5}{2(3.14)(4.0)} \simeq 0.179\ \mathrm{A\,m^{-1}}$$

at P. Both currents enter the page, so their fields circulate clockwise. At P the fields make angles $+30^\circ$ and -30° to the rightward horizontal. The vertical components cancel and horizontal components add:

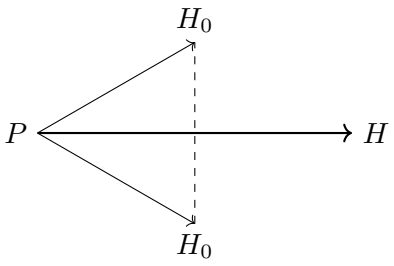
$$H = 2H_0 \cos 30^\circ = \sqrt{3}\, H_0 \simeq 1.73(0.179) = 0.310\ \mathrm{A\,m^{-1}}.$$

Ans. $H = 0.31\ \mathrm{A\,m^{-1}}$.

Explanation

Construct the field directions, not radial arrows Each field is tangent to a circle centered on its wire, not directed along the wire-to-P displacement. The right-hand rule sets the tangent's orientation.

A component construction The following vectors start at the same point; the horizontal arrow is their sum.



Do not insert a permeability into an H answer In vacuum, $B = \mu_0 H$ would convert this result to tesla. Multiplying by μ_0 while retaining $\mathrm{A\,m^{-1}}$ would mix incompatible quantities and units.

6(2) Subtract oppositely directed contributions

Solution

At the center of the circular loop,

$$H_{\text{loop}} = \frac{I}{2r} = \frac{5.0}{2(1.0)} = 2.5 \text{ A m}^{-1}.$$

The counterclockwise loop current produces a field out of the page. At Q, to the right of the upward-current straight wire, that wire's field points into the page and has magnitude

$$H_{\text{straight}} = \frac{5.0}{2(3.14)(3.0)} \simeq 0.265 \text{ A m}^{-1}.$$

Thus

$$H = H_{\text{loop}} - H_{\text{straight}} \simeq 2.235 \text{ A m}^{-1}.$$

Ans. $H = 2.2 \text{ A m}^{-1}$, out of the page.

Explanation

Use a separate right-hand rule for each wire The loop and straight wire have different geometries. Their arrows cannot be compared merely by noting that both carry positive current.

The center-of-loop formula is different For a circular loop $H = I/(2r)$ at its center, whereas an infinite straight wire gives $I/(2\pi r)$. The extra π is not shared.

Check which contribution dominates The loop gives 2.5, much larger than the straight wire's 0.265. The sum must therefore point out of the page with a magnitude slightly below 2.5, not above it.

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Part 25 2017 / 14 / Peroxide titration and electron balance

The official answer uses acidic permanganate reduction to manganese(II). The printed question does not specify the medium; state this assumption because it determines the mole ratio.

14 Match the electron transfers before using the volumes

Solution

In the intended acidic model, the half-reactions are

$$\mathrm{MnO_4^- + 8H^+ + 5}e^- \longrightarrow \mathrm{Mn^{2+} + 4H_2O},$$

$$\mathrm{H_2O_2} \longrightarrow \mathrm{O_2 + 2H^+ + 2}e^-.$$

Multiplying the first by two and the second by five gives

$$\mathrm{2MnO_4^- + 5H_2O_2 + 6H^+} \longrightarrow \mathrm{2Mn^{2+} + 5O_2 + 8H_2O}.$$

The titrant supplies

$$n(\mathrm{MnO_4^-}) = (0.020)(0.0120) = 2.40 \times 10^{-4}\ \mathrm{mol}.$$

Therefore

$$n(\mathrm{H_2O_2}) = \frac{5}{2}(2.40 \times 10^{-4}) = 6.00 \times 10^{-4}\ \mathrm{mol},$$

$$c = \frac{6.00 \times 10^{-4}}{0.0100} = 0.0600\ \mathrm{mol\,L^{-1}}.$$

Ans. Option 2: $0.060\ \mathrm{mol\,L^{-1}}$, for acidic titration.

Explanation

Peroxide is the reducing agent here Oxygen in peroxide changes from -1 to zero in $\mathrm{O_2}$. Peroxide loses electrons while permanganate gains them; the fact that peroxide can oxidize other substances does not reverse its role here.

Use the analyte volume only at the final concentration step The 12.0 mL is titrant volume. Divide the peroxide amount by its original 10.0 mL aliquot, not by the combined endpoint volume.

Why the medium matters In another medium, permanganate can form a different manganese product and accept a different number of electrons. The 5 : 2 peroxide-to-permanganate ratio is not medium-independent.

Check atom and charge balance The combined reaction has 16 H and 18 O atoms on each side. Its total charge is $2(-1) + 6(+1) = +4$ on the left and $2(+2) = +4$ on the right.

Part 29 2017 / 18 / Constitutional and geometrical isomers

The official six-isomer count includes cis/trans isomers. Count carbon connectivities first, then split only the alkene that admits two distinct geometries.

18 Exhaust the one-double-bond and one-ring possibilities

Solution

For a hydrocarbon, the degree of unsaturation is

$$\mathrm{DBE} = \frac{2C + 2 - H}{2} = \frac{2(4) + 2 - 8}{2} = 1.$$

Thus C_4H_8 can have one double bond or one ring. The acyclic connectivities are

$CH_2{=}CHCH_2CH_3$	but-1-ene,
$CH_3CH{=}CHCH_3$	but-2-ene,
$CH_2{=}C(CH_3)_2$	2-methylpropene.

But-2-ene has two different substituents on each double-bond carbon, so it has cis and trans forms. The other two connectivities do not. The acyclic total is therefore $1 + 2 + 1 = 4$.
The remaining connectivities are cyclobutane and methylcyclopropane. In the skeletal drawings below, each unlabeled vertex is carbon and the hydrogens complete carbon valence.

cyclobutane methylcyclopropane

Neither ring introduces an additional cis/trans pair here.

Ans. Option 4: 6 isomers, including geometrical isomers.

Explanation

Why the structural count alone gives five Three acyclic and two cyclic connectivities give five. The two geometries of but-2-ene add one, producing six.

Test each end of the double bond But-1-ene and 2-methylpropene each have a terminal CH_2 group. Two identical H substituents on that carbon prevent a cis/trans distinction.

Avoid counting a reversed name as another compound Reversing the numbering of the four-carbon chain does not create another connectivity. Likewise, rotating a single bond changes a conformation, not the isomer count used here.

Use the unsaturation count to stop the search A ring plus a double bond, or a triple bond, requires two degrees of unsaturation and hence C_4H_6, not C_4H_8. These possibilities are excluded.

Part 40 2018 / 9 / Crystallization with water of hydration

Unlike an anhydrous salt, the pentahydrate removes both dissolved salt and water from the liquid. This five-mark problem therefore requires two changing masses.

9 Subtract both components of the hydrated crystals

Solution

The formula masses are

$$M(\mathrm{CuSO_4}) = 64 + 32 + 4(16) = 160, \qquad M(\mathrm{CuSO_4 \cdot 5H_2O}) = 160 + 5(18) = 250.$$

Initially, the 200 g saturated solution contains

$$m_s = 200\frac{54}{154} = 70.1299\ldots \text{ g}, \qquad m_w = 200\frac{100}{154} = 129.8701\ldots \text{ g}.$$

Let x grams of pentahydrate crystallize. The crystals contain $160x/250 = 0.64x$ grams of anhydrous salt and $90x/250 = 0.36x$ grams of water. The final saturation condition is

$$\frac{m_s - 0.64x}{m_w - 0.36x} = \frac{20}{100}.$$

Consequently,

$$m_s - 0.20m_w = (0.64 - 0.072)x = 0.568x,$$

$$x = \frac{200(54-20)/154}{0.568} = 77.739\ldots \text{ g}.$$

The closest listed value is 77 grams.

Ans. Option 5: 77 g of pentahydrate.

Explanation

The solubility and crystal formulas use different mass bases The solubility describes anhydrous $\mathrm{CuSO_4}$ per mass of water. The requested crystal mass includes five waters per formula unit, so the two quantities cannot be subtracted directly.

Water remaining in solution is not constant About $0.36(77.739) = 27.986$ g of water enters the crystals. Treating the initial water mass as unchanged misses this loss and gives the wrong saturation equation.

Check the final liquid Approximately 20.377 g of salt and 101.884 g of water remain, with ratio 0.200. Their total, 122.261 g, plus the crystals gives the original 200 g, assuming no evaporation.

Select the nearest alternative after calculating The unrounded result is about 77.74, not exactly 77. The printed options request the nearest listed value; they do not require replacing the calculated value by an exact integer identity.

Part 70 2019 / 20 / Empirical versus molecular formula

The percentages give an empirical formula; the stated molecular weight fixes its multiplier. The official key selects the empirical formula instead of the requested molecular formula.

20 Apply both the elemental ratio and molecular weight

Solution

On a 100-gram composition basis, the elemental mole ratio is

$$n_{\mathrm{C}} : n_{\mathrm{H}} : n_{\mathrm{O}} = \frac{54.5}{12} : \frac{9.1}{1} : \frac{36.4}{16} = 4.5417\ldots : 9.1 : 2.275.$$

Dividing by the smallest value gives approximately

$$1.996 : 4.000 : 1.000 \simeq 2 : 4 : 1.$$

The empirical formula is $\mathrm{C_2H_4O}$. Its formula mass and the multiplier required by the given molecular weight 88 are

$$M_{\mathrm{emp}} = 2(12) + 4(1) + 16 = 44, \qquad \frac{88}{44} = 2.$$

The molecular formula is therefore $\mathrm{C_4H_8O_2}$.

Ans. Option 4: $\mathrm{C_4H_8O_2}$, from the printed data. The official key lists option 2, $\mathrm{C_2H_4O}$.

Explanation

Why the two formulas share percentages Multiplying every subscript by two doubles both elemental masses and total molecular mass. The fractions are unchanged, so the percentage data alone cannot distinguish the two formulas.

Check all supplied conditions For $\mathrm{C_4H_8O_2}$, the molecular weight is $48 + 8 + 32 = 88$, with percentages

$$\frac{48}{88}(100) = 54.545\ldots, \quad \frac{8}{88}(100) = 9.091\ldots, \quad \frac{32}{88}(100) = 36.364\ldots.$$

They round to the supplied composition. The empirical formula has the same percentages but molecular weight only 44.

Retain the discrepancy rather than forcing agreement The original question asks for a molecular formula and gives molecular weight 88; the published answer table records option 2. This book reports both and explains why option 4 follows from the printed conditions. This is not presented as an official correction.

Rounded percentages should lead to small integer ratios The carbon ratio 1.996 differs slightly from two because the mass percentages were rounded. Introducing an artificial large formula to make that decimal exact would ignore the data precision.